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19-Tone Theory and Applications

Microtonal music is one of those subjects that has always been around, but few people have ever had the will to investigate it thoroughly. The main reason why more people have not dealt with microtonal music is that there are almost no instruments that allow composers to experiment with it. In spite of all this, the music of many cultures even at the present time employs non-equal-tempered scales, and even Western music did until the eighteenth century, when mathematicians worked out the logarithmic basis of equal temperament.

In this article, the author explains how he became interested in 19-tone equal temperament and how he explored the possible resources available in such a system. This involves creating a chord grammar based on similarity relationships, similar to what he has used in his music written in 12-tone equal temperament. Through these considerations he discovered a particular set of chords that have special properties in terms of their interval content, number of transpositions, and relationships to other chords. Finally, the author explains how he used these properties in the music he composed in this temperament.

Keywords:

equal temperament, pitch, pitch class, numerical representation of pitch classes, pitch structures, normal form representation

19-тоновая теория и её применение

Микротоновая музыка — область, которая существовала всегда, но лишь некоторые люди проявляли желание изучить её глубже. Главная причина этого в том, что инструментов, позволяющих композиторам экспериментировать в этой области, практически нет. Тем не менее в музыке многих различных культур вплоть до наших дней применяются неравноступенные темперации, и даже в европейской музыке они бытовали вплоть до XVIII века, пока математики не выработали логарифмическую основу равноступенной темперации.

В данной статье автор объясняет свой интерес к 19-тоновой равноступенной темперации и как он изучил все возможные достижения, присутствующие в этой системе. Это включает в себя создание гармонической аккордовой грамматики, основанной на соотношениях подобия, схожих с теми, которые присутствовали в его музыке, сочиненной в 12-тоновой равноступенной темперации. Благодаря этим соображениям он обнаружил определённый набор аккордов, обладающих особыми свойствами с точки зрения их интервального содержания, количества транспозиций и соотношений с другими аккордами. В заключение автор объясняет, как он использовал эти свойства в музыке, написанной им в этой темперации.

Ключевые слова:

равноступенная темперация, звуковысотность, звуковысотный класс, числовое представление звуковысотных of pitch structures, intervals, interval content, multiplicative operations, complementary operations, number of distinct forms of a set, total chromatic, trichords, tetrachords, pentachords, hexachords, septachords, arrays, weighted pitch classes, array inclusions.

классов, звуковысотные структуры, представление обычной формы звуковысотных структур, интервалы, интервальный состав, множительные операции, взаимодействующие операции, количество различных форм звукоряда, тотальный хроматизм, трихорды, тетрахорды, пентахорды, гексахорды, септахорды, звукоряды, нагруженные звуковысотные классы, включения звукорядов.

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1. Historical Background

Microtonal music is one of those subjects that has always been around, but few people have ever had the will to investigate it thoroughly. I remember a story once told to me by my colleague at Queens College, Joel Mandelbaum, a microtonal composer who wrote a dissertation on the subject in the 1960s. In researching his dissertation, he came across numerous references to a certain book no one seemed to have. He finally found a copy in the Library of Congress. The book had never been read, and he was the first person to tear the pages apart (the book was a European publication that came with the pages folded and uncut).

The main reason why more people have not dealt with microtonal music is that there are almost no instruments that allow composers to experiment with it. For one thing, the calculations necessary to produce different divisions of the octave are complicated. There are some microtonal instruments, but almost none that can put the span of a different number of notes per octave under the fingers of one hand. The Scalatron, a type of microtonal organ designed many years ago by Motorola, has a complicated system that looks more like a typewriter, with keys going up and down at different angles. Playing it is more like touch typing than keyboard performance. Very few people have used or even seen this instrument, and those who do must suffer its weak collection of timbres, which are more boring than a "soap opera" organ. Ives' quarter-tone music attempted to solve this problem by using two pianos tuned a quarter-tone apart, but this system is limited to subdivisions of the 12-tone scale and very difficult to use for improvisation. The chromatic keyboard historically developed for keyboard instruments is such a part of our conceptualizing of both music itself and Western notation that even modern instruments like synthesizers almost universally use it.

The only practical way to obtain a microtonal instrument at the present time is to tune a synthesizer which allows microtonal tuning (there are only a few, but recently more designers have seen the need for it, although not for the reasons outlined here). This works, but you have to adjust your mind and fingers to the notion that what looks and feels like an octave doesn't sound like it any more. In fact, I can imagine composing microtonal music for these instruments that would have to be written out in conventional notation so that ordinary keyboard players would be able to play it, with the resulting improbable situation that the notation played from would not be the same as that heard, and if the music were played on a conventionally tuned instrument it would be a completely different piece! (Would it still make sense?)

In spite of all this, the music of many cultures even at the present time employs non-equal- tempered scales, and even Western music did until the eighteenth century, when mathematicians worked out the logarithmic basis of equal temperament. I would argue that the music of at least a couple of centuries before that time anticipated equal temperament, and since much of it was vocal, can probably be regarded as based on 12-tone equal temperament. What characterizes much of this music to Western ears is that it sounds "out of tune," while not necessarily unpleasant. The main reason why non-Western cultures employ these scales is that they use musical instruments that are made according to traditional methods that usually predate the knowledge of logarithms. Perhaps it is our cultural superiority that leads us to imagine that this music would "tend" toward equal temperament if the inventors had had such knowledge.

There are two standards according to which all tuning systems have been judged. Most people would only think of the first, the overtone series. The second is the number and size of the steps needed to fill in an interval, such as an octave, although in more practical terms smaller intervals such as a fourth or fifth. Western music struggled with these forces for centuries until the 12-tone scale and Western notation emerged. The evolution of triadic tonality went through analogous processes of development. This is a fascinating subject, and one to which present listeners and thinkers are not necessarily attuned, but it is not my subject at this time.

Comparing tuning systems to the overtone series has led us to judge them by how accurately the intervals of the overtone series, particularly the lower intervals, sound in that system. The 12-tone equal-tempered scale has very good fourths and fifths according to this method. The difference between a pure fifth and an equal-tempered fifth amounts to only 0.445 Hz at middle C. In one sense, this means that the difference between a pure and tempered fifth can be measured in beats, and there will always be beating when fifths are used because the pure intervals will be present in the form of overtones. According to this method of evaluating tuning systems, microtonalists are particularly fond of 12-tone, 19-tone, 31-tone and 51-tone temperaments, because these divisions of the octave produce very good intervals of various kinds: 19-tone temperament produces very excellent thirds and sixths (both major and minor), while 31-tone temperament very good fourths and fifths (even better than 12-tone equal temperament). 51-tone temperament produces very good thirds, fourths, fifths and sixths.

This type of reasoning begs the question of why the overtone series should be used as the standard of comparison. That is indeed an important question, and my background in electronic music gives me a different perspective on this problem than most people who have only read about the overtone series and not used it on a daily basis. Most sounds contain overtones, and their strength and variations in amplitude over the course of the sound produce what we perceive as timbre. Other tones in the scale that are very close to overtones tend to blend in with the lower fundamentals, in extreme cases producing a situation where the higher tone disappears and serves only to modify the timbre of the lower one. Unlike all those psychology books that misdefine consonance as "pleasing," the result is *boring*. This is a fundamental criticism that I feel about music in so-called just intonation, such as that of Harry Partch (whose music is quite interesting for other reasons). It is "so-called" because the music rarely achieves that goal. The ultimate result of a just-tuned major triad would meld into a single tone. Consonance is boring. Dissonances, even minute differences in intonation, but more importantly deviations in tonal characteristics while a sound plays, are *interesting*.

The other reason behind the standardization of the 12-tone equally tempered scale is the number of steps needed to fill in an interval. Our ears have been accustomed to the conventional scales to such an extent that it is probably impossible for us to understand the situation faced by the ancients who had to think about how to fill in a space without the benefit of our knowledge. This is probably a situation that would be greatly helped by instruments that would allow us to improvise. There is no doubt that the squeezing of more and more notes into the space of an octave creates greater complexity and the “messiness” of more tones to worry about.

I am not arguing that music which is live performed is, by any accurate measurement of frequencies, played in tune. Pianos are tuned to a rather complex type of meantone temperament. Woodwind instruments have holes at carefully measured distances to produce the supposedly correct pitch, but changes in breath pressure and mouthpiece placement greatly affect intonation. Brass players literally play overtones of lower notes. String players learn to place their fingers in carefully calculated positions. All players constantly adjust their intonation, supposedly on the basis of what they hear. Accurate measurements have rarely been made of what happens under stressful battle conditions (I hope you can forgive the military analogy), and a widespread mythology exists on the part of both musicians and psychologists about what actually happens.

Of course, the judging of any temperament requires carefully tuned instruments so that we can be sure that we are actually hearing what the result of our calculations says we are listening to. It has always seemed to me that computer music is the only natural medium for microtonal experimentation, and that is what I have used in realizing my own works described below. The only disadvantage in the present context is that computer music makes improvisation practically impossible, since every aspect of what you produce has to be carefully imagined and calculated in advance.

I have been experimenting with the possibilities of 19-tone equal temperament, and in the following discussion I will explain some of the reasons why I have chosen to do what I have done. If you have ever had the feeling that you have only scratched the surface of a subject, these experiments have left me with that feeling raised to a power. The possibilities are enormous, and we are only beginning to understand how to think about the ways of dealing with these materials. Many of the basic ways of thinking about musical materials that we have learned from our previous experiences do not apply here; on the other hand, if you're not careful, you can write music that simply sounds like out-of-tune tonality.

2. Basic Concepts

Many of the basic concepts I explain here are similar to those learned in any basic course about atonal music theory, with some differences. As in 12-tone equal temperament, in 19-tone equal temperament we start with the concepts of *pitch* and *pitch class*; only in this case there are 19 pitch classes and 139 pitches in the seven and one-third octave span of the piano keyboard. We will also define a *pitch class set* as a collection of pitch classes, and while we might want to look at large collections of notes, we will spend most of our time dealing with small ones. (Imagine replacing the idea of the 12-note set, produced by leaning on the keyboard, with that of a 19-note set!) It is also necessary to define *transposition* as a basic similarity relationship where we can hear the same pitch class set moved up or down in the pitch space.

The pitch classes of the 19-tone scale will be designated by the integers 0 to 19, with 0 representing the note we know as C. This makes it easy to calculate the differences in 19-tone "semitones" between pitches. For those who prefer a more familiar form of notation, I propose a system of conventional names for the notes below.

The next higher order concept is that of *interval*, or 2-note pitch set. In 19-tone temperament there are *eighteen* intervals. When we extend this to *interval class* (whereby inversions are counted as part of the same structure, since you are considering the shortest distance between the pitches regardless of octave placement), there are *nine* sets. The next higher order concept is that of *pitch structure*, which includes all PC sets related by transposition. Since 19 is a prime number and there are no intervals that evenly divide the octave, there are 19 transpositions of each pitch structure. According to this line of reasoning, the idea of interval class discussed above is simply that of "pitch structure of size 2". Larger pitch structures can also be examined for their interval content by counting their pitch class subsets of two elements, and when these are tabulated the result is known as the *interval content* of a pitch structure (sometimes called a "vector").

These concepts are simple enough, but before we go on, I must digress and note my differences with other theorists who have made the interval content of a PC set the basic concept rather than the pitch structure, as I have done. In 12-tone temperament, because of the fact that 12 is divisible by 2, 3, 4 and 6, a situation is created whereby some PC sets have the same numbers of intervals of the same type even though they are unrelated by transposition or any other simple relationship. Since there was no obvious relationship, these structures were dubbed to be "Z" related. This is a unique and interesting condition, and it ought to rule against the idea that the interval content be made the basic structure. It is also doubtful that any atonal composers made use of the "Z" relationship since it was not discovered until after their music was written, while it is obvious that they used transposition. Many theorists have the idea that things would be simpler if they could reduce the number of basic elements in some fashion, only in this case it only makes things more confusing. In any case, there are far fewer structures to worry about in 12-tone temperament; we will have to do some neat tricks to reduce the number of elements in 19-tone temperament, and we will do so below.

While I am on the subject of my gripes about other theorists, let me give you another, more serious one: some theorists make a table of pitch class sets, and when a set is encountered in a piece under consideration it is simply designated something like "set 3-12." I think it makes far more sense to use the pitch structure designation of a structure and transposition, such as "0 3 4 (3)" to indicate individual PC sets than references to tables. ("0 3 4" is the pitch structure and the transposition is shown in parentheses or as a subscript.) This is exactly like calling something a "B-flat major triad," which gives the precise structure and transposition, and allows for all structures, even those that don't have common names.

The next basic concept is that of a *normal form* representation for pitch structures. Normal form is the choice of one registral ordering of the set as its proper notation, such as choosing the root position of a major triad over the first or second inversions. Intervals are produced by subtracting the other PCs from a given one modulo 19. The rules for normal form are as follows: First, choose the form that results in the *smallest* overall intervallic span. For example, [0 6 12] could be represented as [0 6 12], [0 7 13], or [0 6 13] (in these examples brackets are placed around the sets for easy identification). If the first step does not produce a unique result, chose the form with the smallest second, third, ... interval. (In addition to [0 3 9 13], the forms of this set include [0 6 10 16], [0 4 10 13] and [0 6 9 15]. The second rule resolves [0 3 9 13] as the form of notation.) Fortunately, in 19-tone temperament there are no sets that divide the octave evenly, such as the diminished seventh chord.

3. Multiplicative Operations

The concepts outlined above would be enough to get started on a meaningfully informed process of musical composition, at least where we would be aware of the most basic aspects of pitch vocabulary. I have gone a further distance, however, because of my desire to employ some of the same methods that I use in my non-microtonal music. Probably the next step for any composer would be to define some basic similarity relationships or transformations that could be applied to pitch class sets. In this context the first idea that usually pops out is that of *inversion*. In my view, inversion of but one aspect of what have been called *multiplicative operations*. These operations are produced by multiplying the elements in the pitch structure by a constant, and have the effect of expanding or contracting the intervals in the set. Inversion is often described as multiplying by –1, which is the same as the number of pitches minus 1, and turns the interval pattern "upside down". (I should clarify that I am employing multiplicative operations as *structural* operations and am not implying any particular manner in which the sets would be used in a musical context, although of course the judicious choice of orderings and registrations can clarify the source of a transformation used.)

In music based on the 12-tone scale, because 12 can be divided equally by 2, 3, 4, and 6, the only multiplicative operations that are usually considered are 1, 5, 7, and 11, corresponding to the cycles of ascending and descending minor seconds and perfect fourths. In 19-tone music there are 18 multiplicative operations, and all of them can produce different structures! (It is helpful to remember that multiplicative operations other than 1, 5, 7, and 11 can even be used in 12-tone music, and may in fact have been employed by some composers before things were investigated in this thorough a manner. [0 1 2 3], for example, can be mapped into [0 2 4 6] by M2 and [0 3 6 9] by M3. M4 and M6 would produce duplications of PCs at the octave and greater intervals, thus producing complications that the composers would have to provide some solution for.)

There are certain basic similarities between all sets that are related by multiplicative operations, designated as "M1, M2, ..., M18," and therefore I have devised a system of type classifications and groupings of pitch structures into families based on their behavior under multiplicative operations. This system greatly reduces the complexity caused by the huge number of distinct forms, and makes the territory seem much more manageable. Before getting to that material, however, it is helpful to explain a few more concepts.

Complementary operations are operations that are "inversional" with respect to one another, or in simpler terms, operations where the sum of the numbers after the "M" is 19. M1 and M18 are complementary, as are M2 and M17, ..., M9 and M10. Pitch structures that are related by complementary operations always have the same interval content, and only those sets have that particular interval content.

The *number of distinct forms* of a set refers to the number of different pitch structures that are produced under the 18 multiplicative operations. While all 19-tone pitch structures have 19 distinct transpositions, and most pitch structures have 18 distinct forms, there are two other possibilities that exist: some sets are *self-inverting* and have only 9 distinct forms, and some highly unusual sets have only 6 distinct forms, replicating themselves under M7 and M11.

The *total chromatic* is, as in 12-tone theory, a representation of all the possible PCs, 19 in this case.

In 12-tone equally tempered music, there are the same number of intervals as there are decachords, trichords as nonachords, tetrachords as octachords, and pentachords as septachords, with hexachords a special case. In 19-tone music, there are the same number of intervals as there are pitch structures of size 17 (I don't think there is a word for that), pitch structures of size 3 as 16, etc. This is mentioned only so that I can describe the *notation* for sets of a large number of elements (more than half the PCs in the system) in terms of the pitch classes that they *exclude*. (This is also useful for 12-tone equal tempered music.) The term "Excl.," when preceding the designation of a PC set, such as "Excl. 0 1 4 6 (9)," refers to the set of PCs *excluding* the designated set, in this case a set of 15 PCs. Thus, in the following table only sets up to size 9 need be considered.

The following table 1 shows the number of pitch structures and the number of families of pitch structures related by multiplicative operations that exist for each 19-tone pitch structure, and for comparison, the number of 12-tone structures of the same sizes. (I have not calculated the sets larger than hexachords.)

Table 1.

Size	12-tone PSs	19-tone PSs	19-tone PS Families
2	6	9	9
3	19	51	4
4	43	204	14
5	66	612	36
6	80	1,428	86
7	66	2,652	154
8	43	3,978	228
9	19	4,862	280

I originally designated families by letter names, in order to use Roman numerals for type classifications and Arabic numbers for PCs and PSs. This produces family names of "A" through "N" for tetrachords, "A" through "JJ" for pentachords, and "A" through "FFFF" for hexachords. This gets to be cumbersome and is probably impractical, but I do not intend to make a thorough investigation of bigger chords.

Type classifications are based on an investigation of the behavior of the sets under the multiplicative operations. While entirely empirical, these are useful in order to know what possibilities may exist with a particular PC set that you may choose to employ. The principles used to form these classifications are the number of distinct forms that the set produces under multiplicative operations and the number of intervals that the sets contain.

Type I:	the set has only six distinct forms.
Type II:	the set has nine distinct forms, complementary operations producing the same form.

All higher-numbered types contain 18 distinct forms. Type classifications are based on the number of interval classes that are present and absent in the forms.

Type III:	only 3 interval classes are present; 6 are absent.
Type IV:	4 ICs are present, 5 are absent.
Type V:	5 Ics are present, 4 are absent.
Type VI:	6 ICs are present, 3 are absent.
Type VII:	7 ICs are present, 2 are absent.
Type VIII:	8 ICs are present, 1 is absent.
Type IX:	all interval classes are present.

The usefulness and interest of the sets increases with the type. Not all PSs of a given size contain all types. Trichords contain only types I, II, and III. Tetrachords contain types I through VI. Pentachords do not contain families of types I or III.

4. 19-tone Intervals

The first thing that it would be useful to do would be to compare the 18 19-tone intervals to both the overtone series and

Table 2.

Step	19-tone Ratio	12-tone Ratio	Interval	Just Interval
0	1.0000	1.0000	Unison	1.0
1	1.0372			
2	1.0757			
3	1.1157			
4	1.1571			
5	1.2001	1.1892	Minor Third	1.2
6	1.2447	1.2599	Major Third	1.25
7	1.2910			
8	1.3389	1.3349	Perfect Fourth	1.3333
9	1.3887			
10	1.4403			
11	1.4938	1.4983	Perfect Fifth	1.5
12	1.5493			
13	1.6069	1.5874	Minor Sixth	1.6
14	1.6666	1.6818	Major Sixth	1.6667
15	1.7285			
16	1.7927			
17	1.8593			
18	1.9284			

to 12-tone equal temperament. Since the semitone and whole tone are really different in 19-tone temperament (there are about 3 19-tone steps for 2 12-tone steps), I don't even bother comparing those (see Table 2).

It can be seen that the intervals listed — major and minor thirds and sixths, and perfect fourths and fifths, have very precise intervals in 19-tone temperament, but there are no corresponding half and whole steps of the 12-tone tempered scale.

It is easy to use the integers 0 through 18 as the names of the notes, and that is what I will do in this document for the most part. I have also used the following system of note names as a mapping of the 19-tone scale onto the five-line staff, using the customary symbols of flats and sharps (see Table 3).

Table 3.

Number	Name	Number	Name
0	C	0	C
1	C#	18	B# or Cb
2	Db	17	B
3	D	16	Bb
4	D#	15	A#
5	Eb	14	A
6	E	13	Ab
7	E# or Fb	12	G#
8	F	11	G
9	F#	10	Gb

The right column reads backwards in order to place complementary notes (with respect to C) opposite each other. It is difficult to accept the notion that the difference between C# and Db is the same as that between C and C#, or that our notational system includes Db, D, and D# all as different notes. We could adopt a system using double sharps and flats, or even sesqui-sharps and -flats as some microtonalists have done. Nevertheless, music transcribed in this notation is easier to follow, and the notes give a pretty accurate picture of where the sounds actually occur. (Of course, notation given to performers on microtonally-tuned synthesizer keyboards such as I described before has to use the note designations for the keys in equal temperament, for that is the only system that the performers know how to play.)

There are some important properties of this notational system. First, all of the intervals that are very close to their 12-tone and just equivalents are transcribed properly: C-Eb is a minor third, C- E a major third, C-F a perfect fourth, C-G a perfect fifth, C-Ab a minor sixth, and C-A a major sixth. Also, C#-E and Db-F are a minor third and major third (intervals of 5 and 6 steps), respectively. Many of the intervals are represented properly, but of course, several are not. Db- E#, for example, is a minor third, and in conventional notation that would represent a doubly augmented second. Allowing the variants of E#-Fb and B#-Cb allow some of these problems to be avoided, but not all. In order to have a correctly notated minor third above Gb, we would need a Bbb, and I have been unwilling to go that far. In this notational system, all the perfect fourths and fifths can be right; it is the thirds and sixths that sometimes look weird.

Can conventional acoustic instruments play in 19-tone temperament? Actually, they can, but they can't play all the notes of the scale without learning how to hear the intervals much more accurately. However, because of the very close intonation of major and minor thirds and sixths and perfect fourths and fifths, the notes C, Eb, E, F, G, Ab and A could be played with acceptable differences in intonation. While the good intervals would work properly beginning from any reference tone on an instrument that could play in adjustable intonation, like a violin or trombone, they would first need to hear a reference tone in order to make the proper adjustment. Instruments with fixed pitches, like woodwinds, piano or pitched percussion, could only play the seven notes listed above, but these would be all right.

5. Pitch Structure Data

These concepts can only be clarified by looking at some of the data. All will be listed by types, groups, and multiplicative operations.

A. Trichords

The following table 4 list the four families of 19-tone trichords. Group A, type II, has only nine distinct forms, and group D, type I, only six.

Table 4.

Group A		Group D	
M1, M18	0 1 2	M1, M7, M11	0 1 8
M2, M17	0 2 4	M2, M3, M14	0 3 5
M3, M16	0 3 6	M4, M6, M9	0 6 10
M4, M15	0 4 8	M5, M16, M17	0 2 5
M5, M14	0 5 10 diminished triad	M8, M12, M18	0 7 8
M6, M13	0 6 12 augmented triad	M10, M13, M15	0 4 10

Table 4 (continued).

Group A		**Group D**	
M7, M12	0 5 12		
M8, M11	0 3 11		
M9, M10	0 1 10		
Group B		**Group C**	
M1	0 1 3	0 1 4	
M2	0 2 6	0 2 8	
M3	0 3 9	0 7 10	
M4	0 7 11	0 3 7	
M5	0 4 9	0 1 5	
M6	0 1 7	0 5 6	
M7	0 2 7	0 7 9	
M8	0 5 8	0 5 11	minor triad
M9	0 8 9	0 8 10	
M10	0 1 9	0 2 10	
M11	0 3 8	0 6 11	major triad
M12	0 5 7	0 2 9	
M13	0 6 7	0 1 6	
M14	0 5 9	0 4 5	
M15	0 4 11	0 4 7	
M16	0 6 9	0 3 10	
M17	0 4 6	0 6 8	
M18	0 2 3	0 3 4	

B. Tetrachords

There are 204 19-tone tetrachords, which can be grouped into 14 families (which I label "A" through "N"). The following (see Table 5) is a summary of the possibilities.

The intervals shown are merely meant to suggest the number of intervals of different types that occur, not the precise locations. In group A, for example (the 0 1 2 3 group), there are always three intervals of one type, two of another and one of a third.

You may be able now to see where I am going with this. The more interesting structures are the ones with both 18 distinct transpositions and the greatest interval count. In this case, they are groups H, K and N (see Table 6).

C. Pentachords

There are 612 19-tone pentachords, which can be sorted into 36 groups labeled "A" through "JJ" (assuming that we label the ones after "Z" as "AA, BB," etc.). Table 7 — is a summary of what they are.

Table 5.

Type	No. Transpositions	Intervals	Groups
I.	6	[111111000]	I
II.	9	[321000000]	A
		[221100000]	E, J, L
IV.	18	[221100000]	B
V.	18	[211110000]	C, D, F, G, M
VI.	18	[111111000]	H, K, N

Table 6.

	Group H	Group K	Group N
M1	0 1 3 7	0 1 4 6	0 2 3 7
M2	0 5 7 11	0 2 8 12	0 5 9 11
M3	0 2 3 9	0 6 7 10	0 2 6 9
M4	0 4 9 12	0 3 7 8	0 1 4 11
M5	0 1 4 9	0 1 5 11	0 5 6 9
M6	0 1 5 7	0 2 7 8	0 6 7 11
M7	0 2 7 11	0 4 7 9	0 3 8 10
M8	0 1 6 9	0 2 5 11	0 2 3 8
M9	0 6 8 9	0 7 8 9	0 1 7 9
M10	0 1 3 9	0 2 3 10	0 2 8 9
M11	0 3 8 9	0 6 9 11	0 5 6 8
M12	0 4 9 11	0 2 5 9	0 2 7 10
M13	0 2 6 7	0 1 6 8	0 4 5 11
M14	0 5 8 9	0 6 10 11	0 3 4 9
M15	0 3 8 12	0 1 5 8	0 7 10 11
M16	0 6 7 9	0 3 4 10	0 3 7 9
M17	0 4 6 11	0 4 10 12	0 2 6 11
M18	0 4 6 7	0 2 5 6	0 4 5 7

Table 7.

Type	No. Transpositions	Intervals	Groups
II.	9	[111111000]	A
		[322210000]	EE
		[222211000]	T, AA

Table 7 (continued).

Type	No. Transpositions	Intervals	Groups
V.	18	[332110000]	B
	18	[322210000]	F
VI.	18	[322111000]	C, G, K, O, Q, HH
	18	[222211000]	FF
VII.	18	[321111100]	D, E, U
		[222111100]	H, I, L, R, V, W, Y, GG, II, JJ
VIII.	18	[221111110]	J, M, N, S, X, Z, BB, CC, DD
IX.	18	[211111111]	P

With pentachords, we see that there is just one group that has all intervals. It is listed below:

Table 8.

	Group P (16)
M1	0 1 2 6 9
M2	0 6 7 9 11
M3	0 1 4 7 9
M4	0 2 6 7 10
M5	0 5 7 10 11
M6	0 4 5 7 13
M7	0 5 9 11 12
M8	0 2 7 8 11
M9	0 2 3 8 12
M10	0 4 9 10 12
M11	0 3 4 9 11
M12	0 1 3 7 12
M13	0 2 3 7 13
M14	0 1 4 6 11
M15	0 3 4 8 10
M16	0 2 5 8 9
M17	0 2 4 5 11
M18	0 3 7 8 9

These sets figure prominently in the 19-tone music I have written.

D. Hexachords

With hexachords, we start dealing with really large amounts of data, which those who are accustomed to using 12-tone chords are unprepared. There are 80 12-tone hexachords, but there are 1,428 19-tone hexachords, which may conveniently be classified into 86 groups. I could designate these “A” through “HHHH”, but I have instead just numbered them. With this huge amount of data, it is quite helpful that we can reduce the forms to just 86 groups to work with.

The following table 9 gives a summary of these forms.

Following the patterns set above, the most interesting hexachords are the ones that contain all intervals with the most balanced distribution, and these would be the last four in the list above (see Table 10).

Table 9.

Type	No. Transpositions	Intervals	Groups
I.	6	[222222111]	33
II.	9	[543210000]	1
		[433221000]	20,72
		[432221100]	36, 52, 79, 85
		[322222110]	80
		[222222111]	81
VI.	18	[443211000]	2
		[433221000]	6
VII.	18	[433211100]	3
		[433211100]	7, 75
		[422222100]	11
		[333221100]	21, 50, 73
		[332222100]	25
		[333311100]	49
VIII.	18	[432211110]	4, 26
		[422221110]	16, 19
		[333211110]	8, 22, 54, 57
		[332221110]	10, 12, 23, 44, 51, 53, 64, 74, 76, 82, 83
		[322222110]	15, 24, 29, 37, 39, 47, 58, 59, 60, 66, 69, 77, 86
		[222222210]	35
IX.	18	[432111111]	5
		[333111111]	84
		[332211111]	9, 13, 17, 38, 48, 68
		[322221111]	14, 18, 27, 28, 30, 31, 34, 40, 41, 43, 45, 46, 55, 62, 63, 65, 67, 70, 71, 78
		[222222111]	32, 42, 56, 61

Table 10.

	Group 32 (FF)	Group 42 (PP)	Group 56 (DDD)	Group 61 (III)
M1	0 1 2 4 7 11	0 1 2 5 8 10	0 1 3 4 8 10	0 1 3 5 8 9
M2	0 5 7 8 9 13	0 3 4 5 7 13	0 3 4 5 9 11	0 2 3 5 9 13
M3	0 2 7 9 10 13	0 4 8 11 13 14	0 3 5 9 11 12	0 4 7 9 12 13
M4	0 3 7 9 11 12	0 1 2 4 8 13	0 1 4 7 9 11	0 1 5 7 8 11
M5	0 1 3 4 8 13	0 2 5 6 10 12	0 3 7 8 9 12	0 4 6 9 10 12

Table 10 (continued).

	Group 32 (FF)	Group 42 (PP)	Group 56 (DDD)	Group 61 (III)
M6	0 4 5 6 9 12	0 3 6 10 11 12	0 1 4 6 7 11	0 4 5 10 12 13
M7	0 2 4 7 12 13	0 6 7 9 11 12	0 1 3 8 10 14	0 2 3 5 9 10
M8	0 4 5 8 10 11	0 3 5 7 10 11	0 4 5 7 8 13	0 4 6 9 11 12
M9	0 1 2 6 8 11	0 2 7 8 11 12	0 1 6 7 9 11	0 4 9 11 12 13
M10	0 3 5 9 10 11	0 1 4 5 10 12	0 2 4 5 10 11	0 1 2 4 9 13
M11	0 1 3 6 7 11	0 1 4 6 8 11	0 5 6 8 9 13	0 1 3 6 8 12
M12	0 1 6 9 11 13	0 1 3 5 6 12	0 2 3 8 12 14	0 1 5 7 8 10
M13	0 3 6 7 8 12	0 1 2 6 9 12	0 4 5 7 10 11	0 1 3 8 9 13
M14	0 5 9 10 12 13	0 2 6 7 10 12	0 3 4 5 9 12	0 1 2 5 7 11
M15	0 1 3 5 9 12	0 5 9 11 12 13	0 2 4 7 10 11	0 3 4 6 10 11
M16	0 3 4 6 11 13	0 1 3 6 10 14	0 1 3 7 9 12	0 1 4 6 9 13
M17	0 4 5 6 8 13	0 2 3 4 7 13	0 2 6 7 8 11	0 4 8 10 11 13
M18	0 4 7 9 10 11	0 2 5 8 9 10	0 2 6 7 9 10	0 1 4 6 8 9

E. Septachords

There are 2652 septachords, not quite double the number of hexachords, and these break down into 154 groups (see Table 11).

As before, the interesting forms are the ones with all intervals distributed in the most even manner. Table 12 shows these groups.

Table 11.

Type	No. Transpositions	Intervals	Groups
I.	6	[333222222]	130, 135
II.	9	[654321000]	1
		[544322100]	136
		[543222210]	151
		[444322110]	101
		[443322210]	62, 152
		[433322220]	154
		[444222111]	149
		[433222221]	92
VIII.	18	various	21 groups
IX.	18	all with [5]	6 groups
		all with [4]	numerous
		all with [3s, 2s, 1s]	33 groups
		[333222222]	89, 118, 149

Table 12.

	Group 89	Group 118	Group 147
M1	0 1 2 5 7 8 11	0 1 3 5 7 10 11	0 2 3 4 8 9 11
M2	0 4 6 9 11 12 13	0 1 2 3 6 10 14	0 2 3 6 7 9 11
M3	0 1 5 7 8 10 11	0 2 5 6 10 12 13	0 1 3 4 7 9 14
M4	0 1 4 6 8 9 13	0 1 2 4 6 9 12	0 2 6 7 10 11 13
M5	0 1 3 5 8 9 13	0 3 4 5 7 12 13	0 2 4 5 6 11 14
M6	0 4 6 9 10 11 12	0 1 4 5 7 10 12	0 4 5 7 11 13 14
M7	0 4 7 9 11 12 13	0 4 6 9 12 13 14	0 4 5 6 7 11 14
M8	0 1 5 9 11 12 14	0 1 3 5 6 9 13	0 1 3 4 7 12 14
M9	0 3 4 8 10 11 13	0 4 6 7 8 9 14	0 2 3 4 8 9 12
M10	0 2 3 5 9 10 13	0 1 2 3 5 9 14	0 3 4 8 9 10 12
M11	0 2 3 5 9 13 14	0 4 7 8 10 12 13	0 2 7 10 11 13 14
M12	0 1 2 4 6 9 13	0 1 2 5 8 10 14	0 3 7 8 9 10 14
M13	0 1 2 3 6 8 12	0 2 5 7 8 11 12	0 1 3 7 9 10 14
M14	0 4 5 8 10 12 13	0 1 6 8 9 10 13	0 1 2 4 6 11 14
M15	0 4 5 7 9 12 13	0 3 6 8 10 11 12	0 2 3 6 7 11 13
M16	0 1 3 4 6 10 11	0 1 3 7 8 11 13	0 2 5 6 8 9 14
M17	0 1 2 4 7 9 13	0 4 8 11 12 13 14	0 2 4 5 8 9 11
M18	0 3 4 6 9 10 11	0 1 4 6 8 10 11	0 2 3 7 8 9 11

F. Larger Chords

While I have investigated the larger chords, through size 9 (and ones larger than those can be handled through their complements), I haven't used them extensively, and the above discussions go far enough ahead for most purposes.

6. Arrays

Arrays are structures pitch class sets where each PC is a member of at least two dimensions at the same time. Two-dimensional arrays are written out as boxes in which the horizontal dimension represents "voices" and the vertical dimension "chords." The designation of a two-dimensional array gives the number of notes in the voices followed by "×" and the number of notes in the chords (i.e., a 5 × 4 array). Arrays of a higher number of dimensions include sets of arrays of a smaller number of dimensions. In my system, I only consider higher numbers of sets of the same sizes: three-dimensional arrays consist of two-dimensional arrays of the same dimensions (i.e., four 5 × 4 arrays), and four-dimensional arrays sets of three-dimensional arrays (i.e. four 4 × 5 × 4 arrays; in practice things rarely get this heavy).

Since arrays involve a lot of duplication, it is clear that the same PC may occur more than once. When it does, it is called a *weighted PC*. Using arrays involves noting the PC content included in the chords and voices, weighted PCs, and the structure of the chords and voices. Since 5 × 4 is 20, this is the smallest size array that can state the total chromatic.

Arrays can be written out either in musical notation or numerically, using the system described above (C = 0, C# = 1, …, B# = 18). Arrays used in compositions are usually structured very carefully to reflect a given set of concerns. I am interested in arrays where all the chords and voices are related by multiplicative operations, where all intervals are present, and where the total chromatic is present. The following is an example:

M1	0 4 17 5	0 2 6 7 (17)	M13
	1 13 16 14	0 1 3 7 (13)	M1
	2 11 18 7	0 3 8 12 (18)	M15
	6 15 9 8	0 2 3 9 (6)	M3
Wt. = 9	9 10 12 3	0 6 7 9 (3)	M16

Chords: 0 1 2 6 9 (0) [M1], 0 6 7 9 11 (4) [M2], 0 3 7 8 9 (9) [M18], 0 2 4 5 11 (3) [M17]

"Wt = 9" indicates that 9 is the only weighted PC. The total chromatic is represented. Both the chords and voices are related under multiplicative operations, and the chords include two pairs of structures represented by complementary operations: M1 and M18 plus M2 and M17. The voices include one pair of complementary operations, M3 and M16. Arrays with this degree of structure, where both chords and voices are related in this manner, are very rare. I had to discover them by running a computer program that went through all the possible permutations of the chords. Here are some others:

M3	0 12 13 15	0 1 3 7 (12)	M1
	3 1 10 4	0 2 3 9 (1)	M3
	6 14 16 2	0 2 7 11 (14)	M7
	18 7 8 5	0 6 8 9 (18)	M9
Wt. = 8	8 11 17 9	0 1 3 9 (8)	M10

Chords: 0 1 4 7 9 (18) [M3], 0 4 5 7 13 (7) [M6], 0 2 5 8 9 (8) [M16], 0 2 3 7 13 (2) [M13]

M4	0 16 11 1	0 5 8 9 (11)	M14
	4 14 7 18	0 4 9 12 (14)	M4
	8 6 15 9	0 2 3 9 (6)	M3
	5 3 17 13	0 4 9 11 (13)	M12
	17 2 10 12	0 2 7 11 (10)	M7

Chords: 0 2 6 7 10 (17) [M4], 0 2 7 8 11 (14) [M8], 0 3 4 8 10 (7) [M15], 0 3 4 9 11 (9) [M11]

M5	0 1 9 6	0 1 6 9 (0)	M8
	5 8 4 13	0 1 4 9 (4)	M5
	10 17 14 16	0 4 6 7 (10)	M18
	11 18 7 2	0 3 8 12 (18)	M15
Wt. = 7	7 12 3 15	0 4 9 12 (3)	M4

Chords: 0 5 7 10 11 (0) [M5], 0 4 9 10 12 (8) [M10], 0 1 4 6 11 (3) [M14], 0 2 3 8 12 (13) [M9]

M7	0 9 5 16	0 3 8 12 (16)	M15
	7 15 17 3	0 2 7 11 (15)	M7
	14 1 12 11	0 1 3 9 (11)	M10
	4 10 6 18	0 5 7 11 (18)	M2
Wt. = 6	6 13 8 2	0 4 6 11 (2)	M17

Chords: 0 5 9 11 12 (14) [M7], 0 1 4 6 11 (9) [M14], 0 1 3 7 12 (5) [M12], 0 5 7 10 11 (11) [M5]

This is a very special set of arrays, all based on the group P pentachords and group H tetrachords. They all have the same relationships as the first array shown above, and in fact they are all multiplicative forms of one another, which is shown by the "M1, M3, M4, M5 and M7" indicated in the upper left.

This sequence of five arrays states all multiplicative forms of the 0 1 2 6 9 pentachord, with M14 appearing twice, just as each array states all 19 pitch classes, with one PC appearing twice. Not all forms of the voice 0 1 3 7 tetrachords are stated, however; M6 and M11 are missing, M1, M4, M7 and M10 appear twice, and M3 appears three times.

These arrays appear verbatim in my composition *Meditation*. The opening of the piece, based on tetrachords, unfolds all 18 intervals of the 19-tone scale in a manner that shows each of the steps, but in such a way that no single interval ever dominates. In fact, the use of as many intervals as possible without duplicating any one of them, particularly the jarring minor seconds, makes a fluid and interesting context.

7. Array Inclusions

As with arrays based on 12-tone temperament, it is possible to have smaller arrays nested inside larger ones. For example, the following trichordal array is included within the array M1 listed above (repeated for convenience):

M1	0 4 17 5	0 2 6 7 (17)	M13
	1 13 16 14	0 1 3 7 (13)	M1
	2 11 18 7	0 3 8 12 (18)	M15
	6 15 9 8	0 2 3 9 (6)	M3
Wt. = 9	9 10 12 3	0 6 7 9 (3)	M16

Chords: 0 1 2 6 9 (0) [M1], 0 6 7 9 11 (4) [M2], 0 3 7 8 9 (9) [M18], 0 2 4 5 11 (3) [M17]

Trichords:

M1	0 4 17 5	0 2 6 7 (17)
	2 11 18 7	0 3 8 12 (18)
	6 15 9 8	0 2 3 9 (6)

Chords: 0 2 6 (0) [M2], 0 7 11 (4) [M4], 0 8 9 (9) [M9], 0 2 3 (5) [M18]

The chords are all from the trichord group B, and the array is a complete subset of the pentachordal array, with the same voice tetrachords.

There are no tetrachordal subsets all the chords of the array that are in the same groups. There are larger arrays, however:

Hexachords:

M1	0 4 17 5	0 2 6 7 (17)	M13
	1 13 16 14	0 1 3 7 (13)	M1
	2 11 18 7	0 3 8 12 (18)	M15
	6 15 9 8	0 2 3 9 (6)	M3
	9 10 12 3	0 6 7 9 (3)	M16
	12 16 6 2	0 4 9 13 (12)	[group L]

Chords: 0 1 2 6 9 12 (0) [M13], 0 6 7 9 11 12 (10) [M7], 0 3 6 10 11 12 (6) [M6], 0 1 3 5 6 12 (2) [M12]

Wts: 0 4 7 10 14 (2)

Here, all of the chords are from group 42 (PP). The added voice tetrachord is, unfortunately, not from the same group as the other voice tetrachords.

Septachords:

M1	0 4 17 5	0 2 6 7 (17)	M13
	1 13 16 14	0 1 3 7 (13)	M1
	2 11 18 7	0 3 8 12 (18)	M15
	6 15 9 8	0 2 3 9 (6)	M3
	9 10 12 3	0 6 7 9 (3)	M16
	17 7 1 11	0 3 9 13 (17)	[group L]
	7 6 11 12	0 1 5 6 (6)	[group J]

Chords: 0 2 3 4 8 9 11 [M1], 0 2 3 6 7 9 11 (4) [M2], 0 2 3 7 8 9 11 (9) [M18], 0 2 4 5 8 9 11 (3) [M17]

Wts: 0 2 7 8 10 13 14 (17)

All of the chords are from group 147. Of particular interest with the septachordal array is the fact that all the voices have the same multiplicative relationships that they do in the pentachordal array.

There are similarly multiplicative forms of these arrays that yield 18 distinct forms with the same relationships between the chords and voices.

8. Conclusions

These considerations show that, while 19-tone harmony is inherently much more complicated than triadic tonality or even the complexities of atonality and serialism, it can still be understood by the same kinds of principles that apply to other music. The amount of data in 19-tone harmony is staggering, and it cannot be dealt with successfully by intuitive methods or experimentation, especially since we do not have musical instruments that allow experimentation with the sounds.

I have tried to employ methods that are similar to what I have used in my other music, namely by bringing together groups of harmonies related by pitch transformations, combining them into larger structures like arrays, and incorporating smaller arrays into larger ones so that there is a coherent relationship between the subcollections, with common-tone properties providing the basic for continuity and succession. Other composers may approach harmony differently and come up with similarly interesting and coherent music.

I have also integrated a preference towards avoiding the intervals that are unusually clashing, like the "minor seconds" 01 and 02, putting them into contexts where at least they are used with octave separations so that they don't sound so jarring. I also have a preference for maximizing diversity, in such a way that structures that include fewer intervals are avoided in favor of passages that contain all interval classes, if possible.

I have written two compositions employing 19-tone equal temperament that are recorded: *Meditation*, my first composition, written in 1993 and released on *Temperamental Music and Created Sounds*, and *19-tone Clusters*, written in 2010 along with its 12-tone tempered companion, *Clusters*, recorded on the album with the same name. Both works have since been uploaded to YouTube.

Meditation, as the title implies, is a slow and contemplative piece, although it has a palindromic shape with accelerations in each section until the mid-point and decelerations afterwards. It begins with a single tone followed by a long melody that unfolds all the intervals of 19-tone equal temperament. The melody has significant beating at times; there is no vibrato used in the entire piece. Beating is a significant property of all equal temperaments; the only way to avoid it is to use just or pure intervals. As the work proceeds, chords of various sizes occur throughout, again employing all possible interval classes. The work ends on a nearly pure major triad, the last tone fading away much as the work began.

19-tone Clusters, the overtones are all clusters of 5-note chords duplicated through three to four octaves above the note. In other words, harmony becomes spectrum. The amplitudes of these components are varied so that they have a kind of "shimmer" moving up and down the spectrum. There are five different ways in which the sounds are introduced into the piece: the basic cluster, a "variegated" cluster, a "whoosh" sound that attacks each of the components separately, a "gong" sound, and a cluster glissando. The piece begins in the middle range and proceeds through several short passages, each emphasizing a different aspect of the sounds, until it reaches a big climax with all instruments being used, and finally concludes quietly, much as it began.

These explore but a minimum of what is possible with these diverse and interesting sounds.

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